Singapore Maths Training

Teach Less, Learn More
An Essential Guide To Singapore Maths

"If we teach today's students as we taught yesterday's, we rob them of tomorrow."

John Dewey
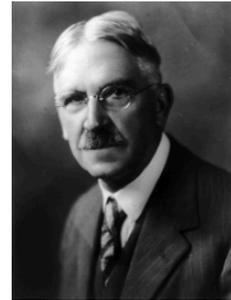
The Maths — No Problem! Combined One-Day Course

MATHS
NO PROBLEM!

Trainer:

Date:

On this course we will:

- Link learning theories to classroom practice

- Identify the core competencies evident in advanced learners

- Explore how lessons are structured

- See how lessons are planned using the online guide

- Model differentiated practice and how it links to assessment

- Identify opportunities to introduce journaling

- Use the textbooks to support learning

- Practice problem solving using more challenging problems

MATHS NO PROBLEM!

Our Journey

TIMELINE

We started by **OFFERING SINGAPORE TEXTBOOKS TO PARENTS** in the UK.

Partnered with the Ark Network to introduce the teaching principles which now underpin their **MATHS MASTERY** programme.

Chosen by the **NCETM** and **DfE** to provide professional development and textbooks to schools in the Maths Hub High Quality Textbook Programme.

2007 **2010** **2011** **2014** **2015** **2016**

We started working with **PIONEERING SCHOOLS** who wanted to emulate the maths results of Singaporean school children.

Published our Primary Maths Series, the first Singapore-style textbooks which are fully aligned with the **2014 ENGLISH NATIONAL CURRICULUM** for mathematics.

Launched our **PRIMARY SERIES ASSESSMENT PAPERS** for teachers to formally test their pupils' mathematical fluency, reasoning and problem-solving skills.

Chosen by the **NCETM** and **DfE** for their High Quality Textbook Programme trial for a third year running.

Why Singapore Maths?

HISTORY OF SINGAPORE MATHEMATICS

UNTIL THE 1980S, SINGAPORE STUDENTS PERFORMED POORLY IN MATHEMATICS

|	1970s	1980s	1990s	2000s
500s	Japan	Hong Kong	Hong Kong	Hong Kong
		Japan	Japan	Japan
		Korea	Korea	Korea
			Singapore	Singapore
400s	Thailand	Philippines	Malaysia	Malaysia
		Singapore	Thailand	Thailand
		Thailand		
300s			Indonesia	Indonesia
			Philippines	Philippines

Source | Hanusek, Jamison, Jamison & Woessmann 2008

TIMSS 4TH GRADE MATH 2015: PERCENTAGE OF STUDENTS AT EACH PROFICIENCY BENCHMARK
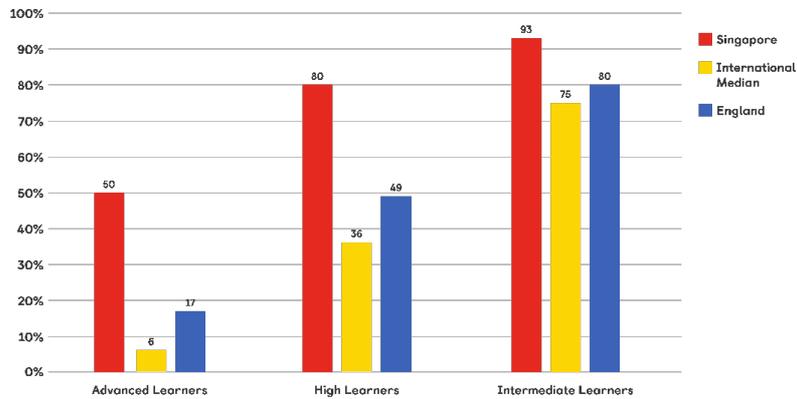
TIMSS 8TH GRADE MATH 2015: PERCENTAGE OF STUDENTS AT EACH PROFICIENCY BENCHMARK
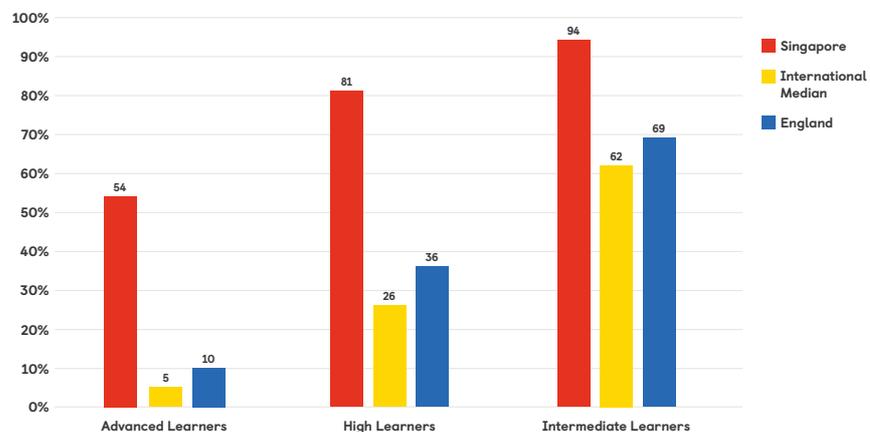
Singapore Mathematics Framework
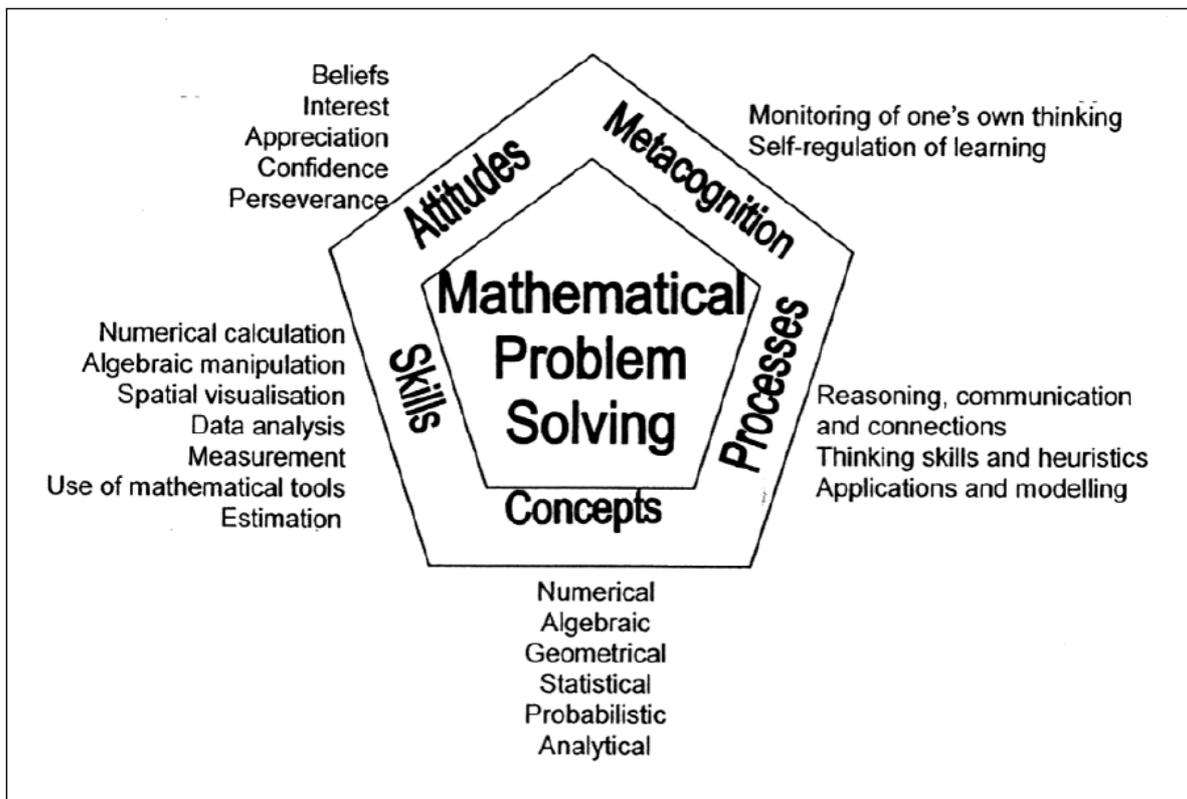
Singapore Ministry of Education (2003)

Key Skills for the 21ˢᵗ Century

Metacognition

Visualisation

Generalisation

Number Sense

Communication

Mastery Learning is a Key Idea in Singapore

Mastery Learning is based on Carroll's model of school learning:

```
                    ┌─────────────────┐
                    │ OPPORTUNITY TO  │
                    │     LEARN       │
                    │ (Time available │
                    │   to learn)     │
                    └─────────────────┘
                    ┌─────────────────┐
                    │   ABILITY TO    │
┌──────────────┐    │   UNDERSTAND    │    ┌──────────────┐
│   APTITUDE   │    │  INSTRUCTION    │    │  ACADEMIC    │
│ (Time needed │    └─────────────────┘    │ ACHIEVEMENT  │
│  to learn)   │    ┌─────────────────┐    └──────────────┘
└──────────────┘    │   QUALITY OF    │
                    │  INSTRUCTION    │
                    └─────────────────┘
                    ┌─────────────────┐
                    │  PERSEVERENCE   │
                    │ (Time student is│
                    │ prepared to spend│
                    │    learning)    │
                    └─────────────────┘
```
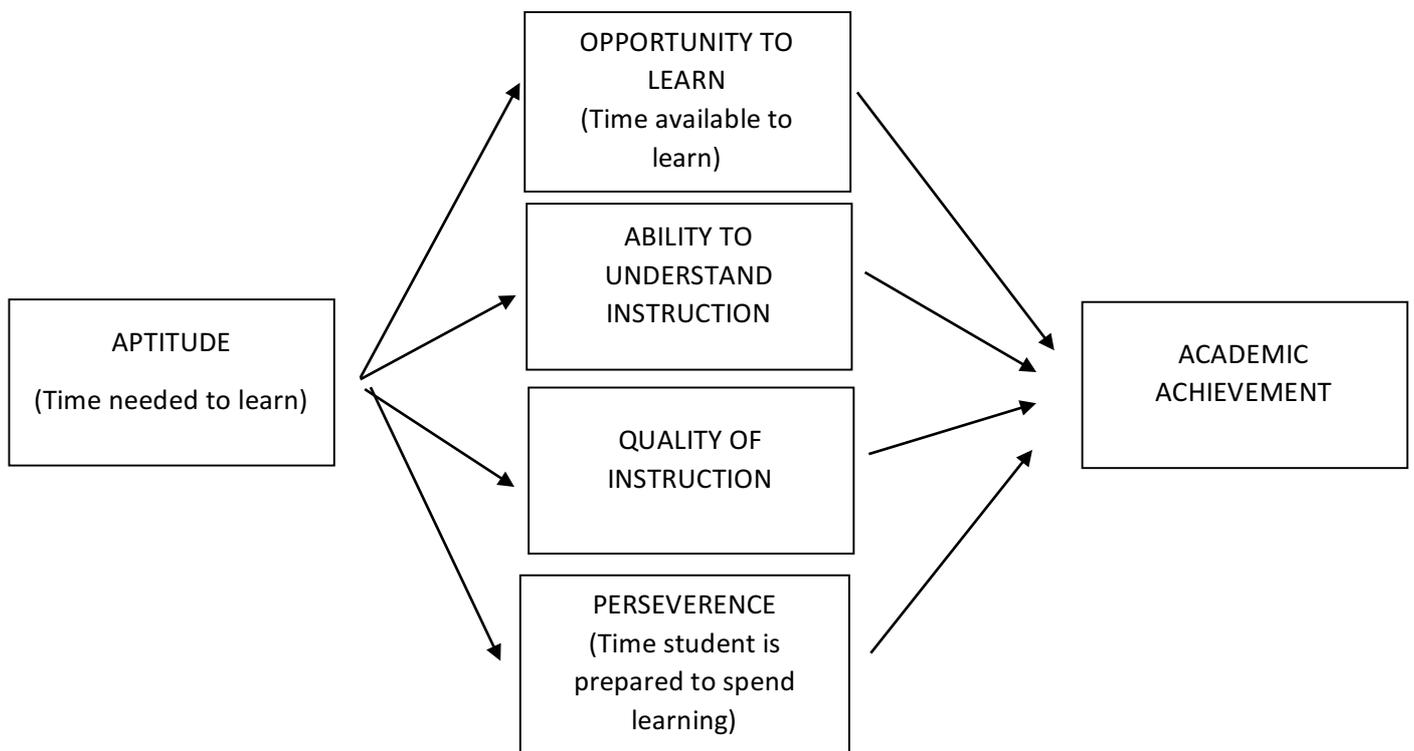
'... aptitude is the length of time it takes a person to learn not how "bright" a person is, i.e. everyone can learn given the right circumstances.'

Benjamin Bloom

Mastery is based on a deep structural understanding of mathematics.

Arthur Baroody described 3 phases of learning number facts:

Phase 1: Counting strategies (using objects or verbal counting)

Phase 2: Reasoning strategies (using known information to determine the unknown)

Phase 3: Mastery (producing answers efficiently – quickly and accurately)

Singapore Maths is Based on Proven Learning Theories

The Singapore maths curriculum was built on 5 key learning theories.

PIAGET

DIENES

VYGOTVSKY

BRUNER

SKEMP

How would you regroup 96 in each case?

Show using a number bond diagram.

96 ÷ 3

96 ÷ 4

96 ÷ 5

96 ÷ 6

96 ÷ 8

1704 ÷ 4

The Structure and Planning of Lessons

1. ## Anchor Task

Each ticket from London to Middlesbrough costs £116.

Adult STD	Ticket Single	Adult One	Price £116.00

From
LONDON
To
MIDDLESBOROUGH

How can we find the cost of 6 tickets from London to Middlesbrough?

2. ## Guided Practice

Guided Practice

1   $135 × 2 =$ ☐

100 10 10 10 1 1 1 1 1

$100 × 2 =$ ☐
$30 × 2 =$ ☐
$5 × 2 =$ ☐
$135 × 2 =$ ☐

3. ## Independent Practice

Worksheet 10

Multiplying 3-Digit Numbers

1   Multiply to find:

(a) $123 × 4 =$ ☐

$100 × 4 =$ ☐

$20 × 4 =$ ☐

$3 × 4 =$ ☐

☐ + ☐ + ☐

= ☐

Dufour's 4 Critical Questions

1. What do we want our children to learn? (Expectations)

2. How will we know they are learning? (Assessment)

3. What if they are not learning? (Intervention)

4. What if they already know it? (Challenge)

Which of these objectives would you pick as your all / most / some expectations?

Year 3: Subtraction
Formative Assessment

- Pupils can subtract single-digit numbers from 3-digit numbers.
- Pupils can subtract single- and double-digit numbers.
- Pupils can subtract double-digit numbers from 3-digit numbers.
- Pupils can subtract the 3-digit numbers from 3-digit numbers.
- Pupils can use number bonds to add and subtract by breaking numbers apart.
- Pupils can use fact families to solve addition and subtraction facts.
- Pupils can use concrete materials and pictorial representations to support subtracting.
- Pupils can use a number line to subtract numbers and count backwards.
- Pupils can use number squares to subtract numbers and count backwards.
- Pupils can use Base 10 materials to represent numbers and subtract them.
- Pupils can count backwards in ones.
- Pupils can use a number bond diagram to subtract the ones, tens and hundreds.
- Pupils can use the column methods for subtraction as a key strategy.
- Pupils can rename 1 ten as 10 ones.
- Pupils can use concrete materials to rename 1 ten as 10 ones.
- Pupils can use concrete materials to rename 1 hundred as 10 tens.
- Pupils can use the number bond strategy as a key strategy to subtracting with renaming.
- Pupils can use the number bond strategy to subtract 3-digit numbers from multiples of 100.

Differentiated Practice

How does the runway model help assessment?

Can you list 5 ways 'struggling learners' are supported during lessons?

1.

2.

3.

4.

5.

Challenging Advanced Learners

Creative Challenge	Conceptual Challenge	Communication Challenge
• Write your own problem • Invent a method • Invent a game	• Show a physical model • Make a drawing • Deeper understanding through connections • Making rules/generalisations	• Explain orally • Explain in writing/make a note • Correct a mistake

Lessons as Learning

What are the 5 elements of learning?

E

S

J

R

P

Different Types of Journaling

Descriptive Journaling — Pupils describe the methods they have used. "Can you write a set of instructions for a friend so that they can solve this problem?"

Evaluative Journaling — Pupils have to make and justify choices. "Why did you choose those methods? How were they helpful?"

Creative Journaling — Pupils are encouraged to develop their own methods/stories "Can you write a story to go with this problem? Can you invent a new method?"

Investigative Journaling — Pupils record their findings after exploring a problem "Did you see any patterns? What helped you in this investigation?"

Bar Modelling

Steps in Bar Modelling

L

D

Q

E

A

Sam has 45 stamps. His father gives him 35 more. How many stamps does Sam have altogether?

Hannah baked 400 tarts.
She gave 270 tarts away.
How many tarts did Hannah have left?

Key Learning Using the Part-Whole Model	**Key Learning Using the Comparison Model**
• Identify equal and unequal parts • Unequal parts = addition and subtraction • Equal parts = multiplication and division	• Identify the base (what is the same) • Identify the difference • Use addition or subtraction to find the difference • Use multiplication or division to work out the base

Jordan spent 1/5 of his pocket money on sweets and 1/2 of the remainder on a book. The book cost £12. How much pocket money did Jordan have?

Jordan spent 1/4 of his pocket on a gift and 1/2 of the remainder on a book. The book cost £12. How much pocket money did Jordan have?

In Focus

At first, 👧 and 👦 had the same amount of money. After 👧 gave 👦 £732, 👦 had 4 times as much money as 👧 had.

How much money did 👧 and 👦 have altogether?

Andrew had £93. With this money, he could buy 3 CDs and 4 books. However, he bought only 2 CDs and 3 books and had £27 left. What was the cost of a CD?

Tracy is 3 times as old as Sam. Sam is 6 years younger than Tracy. How old is their mother if she is 4 times as old as Tracy?

2 pencils and 3 pens cost £18.80. 6 pencils and 6 pens cost £45.
How much is each pen?
How much more does a pen cost than a pencil?

Mr Singh bought some furniture for £1760. This was 20% less that the normal price. He later sold the same furniture for 5% more than the normal price.
How much did Mr Singh sell the furniture for?

At the fair, there were half as many men as women. The number of adults was a third that of children. There were 3600 people at the fair. How many children were there?

The price of a sandwich is 50p more than the price of a drink. The price of a pizza is twice the price of a drink. Two pizzas cost £1.90 more than a drink and a sandwich. Find the price of a drink.

Sam has £30 more than Terry at first.
Terry gave Sam £12. Sam now has 3 times as much money as Terry.
How much money did Terry have at first?

There were 88 children at the school fete. Boys and girls were asked to pair up for the three-legged race. 1/3 of the boys paired up with 2/5 of the girls. The remainder of the children did not participate.
How many boys were at the fete?
How many girls joined in the race?

A computer game costs twice as much as a soft toy. The soft toy costs twice as much as a board game. Their total cost is £224. Find the cost of the soft toy.

10, 12, 14 and 16 are consecutive numbers that add up 52. Another four consecutive numbers add up to 92. Find the largest of these numbers.

Add 8 and 236.

Method 1

Number Sense
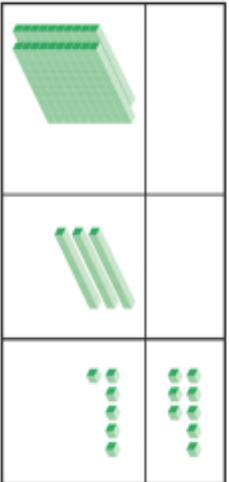
Step 1   Add the ones.

$8 + 6 = 14$

h	t	o
2	3	8
+		
	1	4

Step 2   Add the tens.

$0 + 30 = 30$

h	t	o
2	3	8
+		
	1	4
	3	0

Step 3   Add the hundreds.

$0 + 200 = 200$

h	t	o
2	3	8
+		
	1	4
	3	0
2		

Step 4   Add 14, 30 and 200

$8 + 236 = 244$

h	t	o
2	3	8
+		
	1	4
	3	0
2	0	0
2	4	4

15

Method 2

Step 1 Add the ones.
8 ones + 6 ones = 14 ones

> $8 + 6 = 14$
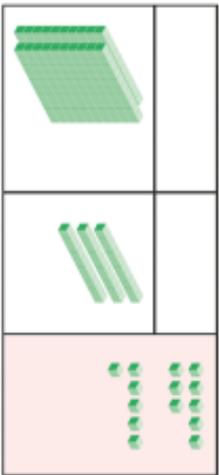
h	t	o
2	3	8
+		6
		4

Step 2 Regroup the ones.
14 ones = 1 ten + 4 ones

$$14 \to 10,\ 4$$
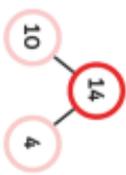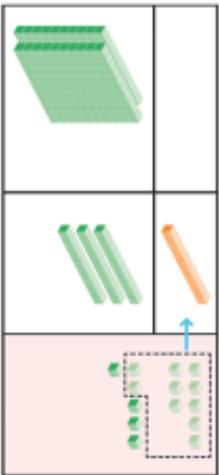
h	t	o
2	3	8
+		6
	1	
4		

Step 3 Add the tens.
1 ten + 3 tens = 4 tens
Add the hundreds.
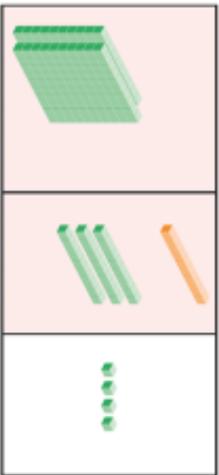
h	t	o
2	3	8
+		6
2	4	4

8 + 236 = 244
There are 244 children altogether.

Method 3

8 + 236 = 244

> $8 + 236 = 4 + 240$
> $8 \to 4,\ 236 \to 4$

Add three numbers

1 Add 7, 3 and 2.

Method 1   Make 10.

$7 + 3 + 2 = 10 + 2$
$= 12$

"7 and 3 make 10."

Method 2   Add by counting on.

7	8	9	10	11	12
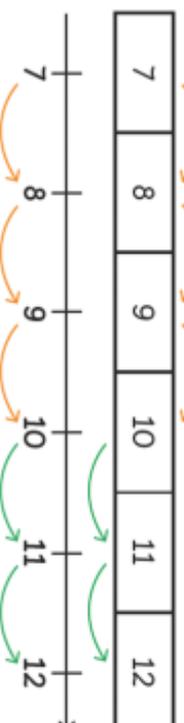
$7 + 3 + 2 = 12$

2 Add 9, 9 and 8.

$9 + 9 + 8 = 26$

$9 + 9 = 18$
$18 + 8 = 26$

17

Add three numbers

Guided Practice

1   Make 10 and add.

(a)   2 + 8 + 4 = ☐   +   ☐

         = ☐

(b)   3 + 9 + 1 = ☐   +   ☐

         = ☐

2   Add.

(a)   6 + 7 + 4 = ☐

(b)   9 + 0 + 4 = ☐

(c)   8 + 5 + 9 = ☐

(d)   7 + 9 + 6 = ☐

Worksheet 13

Addition of Three Numbers

1 Add by counting on.

(a) $5 + 9 + 8 =$ ☐

(b) $8 + 7 + 6 =$ ☐

(c) $16 + 5 + 9 =$ ☐

(d) $5 + 12 + 3 =$ ☐

(e) $28 + 8 + 6 =$ ☐

(f) $9 + 10 + 13 =$ ☐

(g) $30 + 21 + 3 =$ ☐

(h) $3 + 45 + 7 =$ ☐

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60

2 Make 10 and add.

(a) $3 + 9 + 7 =$ ☐ ☐ + ☐

(b) $4 + 2 + 8 =$ ☐ ☐ + ☐

(c) $5 + 7 + 5 =$ ☐

(d) $4 + 9 + 6 =$ ☐

(e) $8 + 3 + 7 =$ ☐

3 Add.

(a) $5 + 8 + 7 =$ ☐

(b) $6 + 7 + 5 =$ ☐

(c) $4 + 3 + 9 =$ ☐

(d) $4 + 9 + 7 =$ ☐

(e) $6 + 3 + 8 =$ ☐

(f) $8 + 8 + 8 =$ ☐

4 Match.

$4 + 5 + 7$ ●	● 15
$3 + 4 + 5$ ●	● 12
$7 + 8 + 9$ ●	● 21
$5 + 5 + 5$ ●	● 24
$9 + 4 + 8$ ●	● 18
$1 + 8 + 9$ ●	● 16

Lesson Structure

Dividing a Fraction by a Whole Number

In Focus

and share

$\frac{3}{4}$ of a cake equally.

Let's Learn

Explain how much of a whole cake each of them gets.

1

$\frac{3}{4} = \frac{6}{8}$

$\frac{3}{4} \div 2 = \frac{6}{8} \div 2$

$= \frac{3}{8}$

2

$\frac{3}{4}$
$\frac{2}{4}$
$\frac{1}{4} = \frac{2}{8}$

$\frac{3}{4} \div 2 = \frac{1}{4} + \frac{1}{8}$

$= \boxed{}$

3
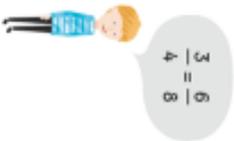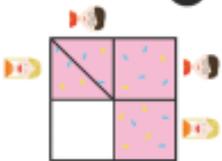
Each person gets half of $\frac{3}{4}$.

$\frac{3}{4} \div 2 = \frac{1}{2} \times \frac{3}{4}$

$= \boxed{}$

Guided Practice

1 Find the value of each.

(a) $\frac{6}{7} \div 2$

(b) $\frac{1}{3} \div 3$

(c) $\frac{2}{5} \div 4$

Name: _____ Class: _____ Date: _____

Dividing a Fraction by a Whole Number

1 Divide.

(a) $\dfrac{4}{7} \div 3 = \dfrac{1}{7} + \dfrac{1}{21}$

$= \dfrac{3}{21} + \dfrac{1}{21}$

= [ ]

$\dfrac{4}{7}$ → $\dfrac{3}{7}$, $\dfrac{1}{7} = \dfrac{3}{21}$

(b) $\dfrac{6}{7} \div 5 =$

(c) $\dfrac{5}{9} \div 4 =$

2 Divide.

(a) $\dfrac{3}{4} \div 9$

$= \dfrac{1}{9} \times \dfrac{3}{4}$

= [ ]

(b) $\dfrac{3}{7} \div 2$

=

(c) $\dfrac{2}{3} \div 10$

=

(d) $\dfrac{4}{5} \div 3$

=

3 Divide.

(a) $\dfrac{3}{4} \div 4$

$= \dfrac{1}{4} \times \dfrac{3}{4}$

= [ ]

(b) $\dfrac{5}{6} \div 4$

=

(c) $\dfrac{2}{7} \div 3$

=

(d) $\dfrac{7}{12} \div 5$

=

21

Struggle is Good! I Want to Fly!

Once a little boy was playing outdoors and found a fascinating caterpillar. He carefully picked it up and took it home to show his mother. He asked his mother if he could keep it, and she said he could if he would take good care of it.

The little boy got a large jar from his mother and put plants to eat, and a stick to climb on, in the jar. Every day he watched the caterpillar and brought it new plants to eat.

One day the caterpillar climbed up the stick and started acting strangely. The boy worriedly called his mother who came and understood that the caterpillar was creating a cocoon. The mother explained to the boy how the caterpillar was going to go through a metamorphosis and become a butterfly.

The little boy was thrilled to hear about the changes his caterpillar would go through. He watched every day, waiting for the butterfly to emerge. One day it happened, a small hole appeared in the cocoon and the butterfly started to struggle to come out.

At first the boy was excited, but soon he became concerned. The butterfly was struggling so hard to get out! It looked like it couldn't break free! It looked desperate! It looked like it was making no progress!

The boy was so concerned he decided to help. He ran to get scissors, and then walked back (because he had learned not to run with scissors...). He snipped the cocoon to make the hole bigger and the butterfly quickly emerged!

As the butterfly came out the boy was surprised. It had a swollen body and small, shriveled wings. He continued to watch the butterfly expecting that, at any moment, the wings would dry out, enlarge and expand to support the swollen body. He knew that in time the body would shrink and the butterfly's wings would expand.

But neither happened!

The butterfly spent the rest of its life crawling around with a swollen body and shriveled wings.

It never was able to fly...

As the boy tried to figure out what had gone wrong his mother took him to talk to a scientist from a local college. He learned that the butterfly was SUPPOSED to struggle. In fact, the butterfly's struggle to push its way through the tiny opening of the cocoon pushes the fluid out of its body and into its wings. Without the struggle, the butterfly would never, ever fly. The boy's good intentions hurt the butterfly.

As you go through school, and life, keep in mind that struggling is an important part of any growth experience. In fact, it is the struggle that causes you to develop your ability to fly.